Solution2(a):

The necessary condition for the space-probe to escape from the Solar system is that the sum of its kinetic and potential energy in the Sun's gravitational field is larger than or equal to zero:

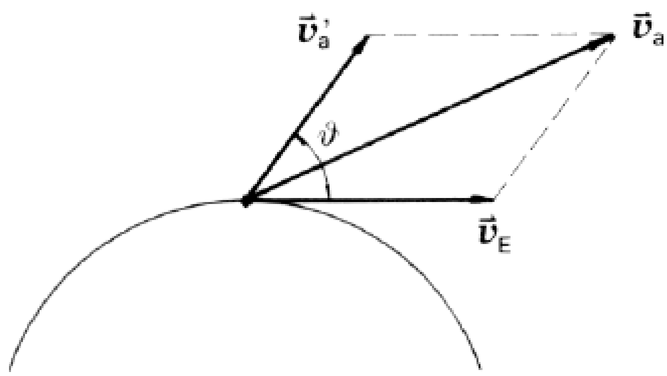
$$\frac{1}{2}mv_a^2 - \frac{GmM}{R_E} \geq 0\,,$$

where m is the mass of the probe, v_a its velocity relative to the Sun, M the mass of the Sun, R_E the distance of the Earth from the Sun and G the gravitational constant. Using the expression for the velocity of the Earth, $v_E = (GM/R_E)^{1/2}$, we can eliminate G and M from the above condition:

$$v_a^2 \geq \frac{2GM}{R_E} = 2v_E^2\,.$$

Let v_a' be the velocity of launching relative to the Earth and ϑ the angle Between $\vec{v}_E$ and $\vec{v}_a'$ (see the Figure).

Then from $\vec{v}_a = \vec{v}_a' + \vec{v}_E$ and $v_a^2 = 2v_E^2$ it



follows:

$$v_a'^2 + 2v_a' v_E \cos\vartheta - v_E^2 = 0$$

and

$$v_a' = v_E\left[-\cos\vartheta + \sqrt{1+\cos^2\vartheta}\right].$$

(because –ve sign is not admissible.)

The minimum velocity is obtained for $\vartheta = 0$:

$$v_a' = v_E(\sqrt{2} - 1) = 12.3 \text{ km/s}\,.$$

Solution 2(b):

The blocks slide relative to the prism with accelerations $\mathbf{a}_1$ and $\mathbf{a}_2$, which are parallel to its sides and have the same magnitude a (see Fig.1). The blocks move relative to the earth with accelerations:

$$\mathbf{w}_1 = \mathbf{a}_1 + \mathbf{a}_0 \quad (1)$$
$$\mathbf{w}_2 = \mathbf{a}_2 + \mathbf{a}_0 \quad (2)$$

because the $\mathbf{a_0}$ is the acceleration of the prism.
Now we project $\mathbf{w}_1$ and $\mathbf{w}_2$ along the *x*- and *y*-axes:

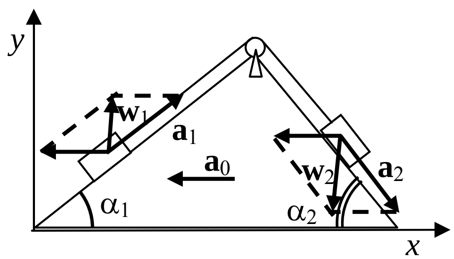


Fig.1

$$w_{1x} = a\cos\alpha_1 - a_0 \quad (3)$$
$$w_{1y} = a\sin\alpha_1 \quad (4)$$
$$w_{2x} = a\cos\alpha_2 - a_0 \quad (5)$$
$$w_{2y} = -a\sin\alpha_2 \quad (6)$$

The equations of motion for the blocks and for the prism have the following vector forms (see Fig.2):

$$m_1\mathbf{w}_1 = m_1\mathbf{g} + \mathbf{R}_1 + \mathbf{T}_1 \quad (7)$$
$$m_2\mathbf{w}_2 = m_2\mathbf{g} + \mathbf{R}_2 + \mathbf{T}_2 \quad (8)$$
$$M\mathbf{a}_0 = M\mathbf{g} - \mathbf{R}_1 - \mathbf{R}_2 + \mathbf{R} - \mathbf{T}_1 - \mathbf{T}_2 \quad (9)$$

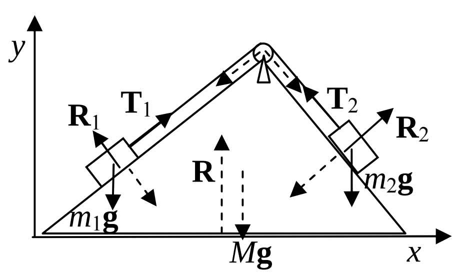


Fig.2

The forces of tension $\mathbf{T}_1$ and $\mathbf{T}_2$ at the ends of the thread are of the same magnitude *T* since the masses of the thread and that of the pulley are negligible. Note that in equation (9) we account for the net force $-(\mathbf{T}_1 + \mathbf{T}_2)$, which the bended thread exerts on the prism through the pulley**.** The equations of motion result in a system of six scalar equations when projected along *x* and *y*:

$$m_1 a\cos\alpha_1 - m_1 a_0 = T\cos\alpha_1 - R_1\sin\alpha_1 \quad (10)$$
$$m_1 a\sin\alpha_1 = T\sin\alpha_1 + R_1\cos\alpha_1 - m_1 g \quad (11)$$

$$m_2 a\cos\alpha_2 - m_2 a_0 = -T\cos\alpha_2 + R_2\sin\alpha_2 \quad (12)$$
$$m_2 a\sin\alpha_2 = T\sin\alpha_2 + R_2\cos\alpha_2 - m_2 g \quad (13)$$
$$-Ma_0 = R_1\sin\alpha_1 - R_2\sin\alpha_2 - T\cos\alpha_1 + T\cos\alpha_2 \quad (14)$$
$$0 = R - R_1\cos\alpha_1 - R_2\cos\alpha_2 - Mg \quad (15)$$

By adding up equations (10), (12), and (14) we get,

$$m_1 a\cos\alpha_1 - m_1 a_0 + m_2 a\cos\alpha_2 - m_2 a_0 - Ma_0$$
$$= T\cos\alpha_1 - R_1\sin\alpha_1 - T\cos\alpha_2 + R_2\sin\alpha_2 + R_1\sin\alpha_1 - R_2\sin\alpha_2 - T\cos\alpha_1 + T\cos\alpha_2$$

Or, $$a(m_1\cos\alpha_1 + m_2\cos\alpha_2) = a_0(m_1 + m_2 + M)$$

Therefore, $$a = a_0\frac{M + m_1 + m_2}{m_1\cos\alpha_1 + m_2\cos\alpha_2} \quad (16)$$

The straightforward elimination of the unknown forces gives the final answer for a_0:

$$a_0 = \frac{(m_1\sin\alpha_1 - m_2\sin\alpha_2)(m_1\cos\alpha_1 + m_2\cos\alpha_2)}{(m_1 + m_2 + M)(m_1 + m_2) - (m_1\cos\alpha_1 + m_2\cos\alpha_2)^2}. \quad (17)$$

It follows from equation (17) that the prism will be in equilibrium ($a_0 = 0$) if:

$$\frac{m_1}{m_2} = \frac{\sin\alpha_2}{\sin\alpha_1}. \quad (18)$$

Solution 3(a) : We use the following notation:

t	temperature of the final equilibrium state,
$t_0 = 0^0$C	the melting point of ice under normal pressure conditions,
M_2	final mass of water,
M_3	final mass of ice,
$m'_2 \le m_2$	mass of water, which freezes to ice,
$m'_3 \le m_3$	mass of ice, which melts to water.

a) Generally, four possible processes and corresponding equilibrium states can occur:

(i). $t_0 < t < t_2$, $m'_2 = 0$, $m'_3 = m_3$, $M_2 = m_2 + m_3$, $M_3 = 0$.
Unknown final temperature t can be determined from the equation
$(m_1c_1 + m_2c_2)(t_2 - t) = m_3c_3(t_0 - t_3) + m_3l + m_3c_2(t - t_0)$ (1)
However, only the solution satisfying the condition $t_0 < t < t_2$ does make physical sense.

(ii). $t_3 < t < t_0$, $m'_2 = m_2$, $m'_3 = 0$, $M_2 = 0$, $M_3 = m_2 + m_3$.
Unknown final temperature t can be determined from the equation
$m_1c_1(t_2 - t) + m_2c_2(t_2 - t_0) + m_2l + m_2c_3(t_0 - t) = m_3c_3(t - t_3)$ (2)
However, only the solution satisfying the condition $t_3 < t < t_0$ does make physical sense.

(iii). $t = t_0$, $m'_2 = 0$, $0 \le m'_3 < m_3$, $M_2 = m_2 + m'_3$, $M_3 = m_3 - m'_3$.
Unknown mass m'_3 can be calculated from the equation

$$(m_1c_1 + m_2c_2)(t_2 - t_0) = m_3c_3(t - t_3) + m'_3 l \qquad (3)$$

However, only the solution satisfying the condition $0 \le m'_3 \le m_3$ does make physical sense.

(iv). $t = t_0$, $0 \le m'_2 \le m_2$, $m'_3 = 0$, $M_2 = m_2 - m'_2$, $M_3 = m_3 + m'_2$.
Unknown mass m'_2 can be calculated from the equation

$$(m_1c_1 + m_2c_2)(t_2 - t_0) + m'_2 l = m_3c_3(t_0 - t_3) \qquad (4)$$

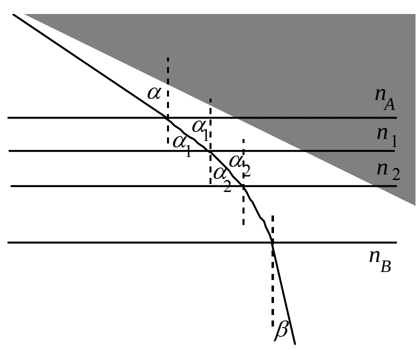
However, only the solution satisfying the condition $0 \le m'_2 \le m_2$ does make physical sense.

b) Substituting the particular values of m_1, m_2, m_3, t_2 and t_3 to equations (1), (2) and (3) one obtains solutions not making the physical sense (not satisfying the above conditions for t, respectively m'_3). The real physical process under given conditions is given by the equation (4) which yields

$m'_2 = [m_3c_3(t_0 - t_3) - (m_1c_1 + m_2c_2)(t_2 - t_0)]/l$

Substituting given numerical values one gets $m'_2 = 0.11$ kg. Hence, $t = 0^0$C, $M_2 = m_2 - m'_2 = 0.89$ kg, $M_3 = m_3 + m'_2 = 2.11$ kg.

Solution 3(b) :



Let us divide the plate in a large number of small slides as shown in the figure such that one can assume that the light ray travels in a straight line within each small slide n_1, n_2, ……. ,n_m so that one can take refractive index constant within the small slide.
Then from the figure, we get,

$$n_A \sin \alpha = n_1 \sin \alpha_1 = n_2 \sin \alpha_2 = \ldots = n_B \sin \beta$$

Or, $n_A \sin\alpha = n_B \sin\beta$.

Solution 4(a) : A circuit equivalent to the given one is shown in Fig.3. In a steady state (the capacitors are completely charged already) the same current I flows through all the resistors in the closed circuit ABFGHDA. From the Kirchhoff's second rule we obtain:

$$I = \frac{E_4 - E_1}{4R}. \qquad (1)$$

Next we apply this rule for the circuit ABCDA:

$$V_1 + IR = E_2 - E_1, \qquad (2)$$

where V_1 is the potential difference across the capacitor C_1. By using the expression (1) for I, and the equation (2) we obtain:

$$V_1 = E_2 - E_1 - \frac{E_4 - E_1}{4} = 1\,\text{V}. \qquad (3)$$

Similarly, we obtain the potential differences V_2 and V_4 across the capacitors C_2 and C_4 by considering circuits BFGCB and FGHEF:

$$V_2 = E_4 - E_2 - \frac{E_4 - E_1}{4} = 5\,\text{V}, \qquad (4)$$

$$V_4 = E_4 - E_3 - \frac{E_4 - E_1}{4} = 1\,\text{V}. \qquad (5)$$

Finally, the voltage V_3 across C_3 is found by applying the Kirchhoff's rule for the outermost circuit IJEHLKDAI:

$$V_3 = E_3 - E_1 - \frac{E_4 - E_1}{4} = 5\,\text{V}. \qquad (6)$$

The total energy of the capacitors is expressed by the formula:

$$W = \frac{C}{2}\left(V_1^2 + V_2^2 + V_3^2 + V_4^2\right) = 26\,\mu\text{J}. \qquad (7)$$

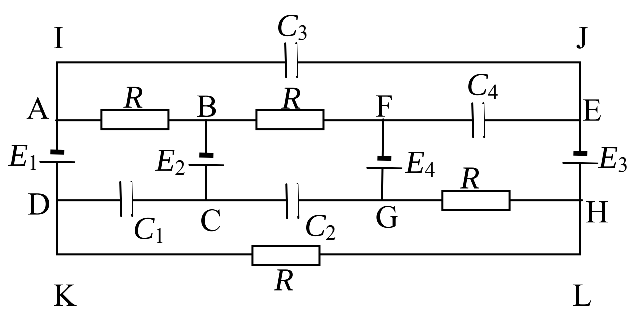


Fig. 3

When points B and H are short connected (circuited) the same electric current I' flows through the resistors in the BFGH circuit. It can be calculated, again by means of the Kirchhoff's rule, that:

$$I' = \frac{E_4}{2R}. \qquad (8)$$

The new steady-state voltage on C_2 is found by considering the BFGCB circuit:

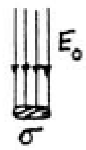
$$V_2' + I'R = E_4 - E_2 \qquad (9)$$

or finally:

$$V_2' = \frac{E_4}{2} - E_2 = 0\,\text{V}. \qquad (10)$$

Therefore the charge q_2' on C_2 in the new steady state is zero.

Solution 4(b) : (i) By Gauss's law, $\sigma = \varepsilon_0 E_0$, for a plane surface,



where σ is the Earth's surface charge density, ε_0 is the permittivity of the free space and E_0 is the electric field at the Earth's surface. Therefore, $\sigma = -8.85*10^{-12} * 150$

$= -1.33 * 10^{-9}$ C/m^2. (1)

And total charge carried on the Earth's surface (Q) = $4\pi R^2\sigma$
where R is the radius of the Earth. Therefore, using (1), we get,
$Q = -4\pi * (6.4*10^6)^2 * 1.33* 10^{-9}$ C $\approx -6.7*10^5$ C.

(ii) Consider a cylinder of cross section A with faces at heights of 0 and 100 m.
By Gauss's law, $E(0)\,A - E(100)\,A = q_{enclosed}/\varepsilon_0 = \rho_{ave.} * (100A)/\varepsilon_0$

Therefore, $\rho_{ave.} = \varepsilon_0\,[E(0)-E(100)]/100$
$= 8.85*10^{-12}\,[150 - 100]/100 \approx 4.43 * 10^{-12}$ C/m^3